S–3873

Sub. Code
23MMA1C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

ALGEBRAIC STRUCTURES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define the conjugate class of a in G.

2. Prove that $a \in z$ if and only if $N(a) = G$.

3. Define the internal direct product of groups.

4. Define an R–module.

5. What is meant by similar linear transformations?

6. Define the index of nilpotent of T.

7. Define the Jordan canonical form.

8. Define the rational canonical form of T.

9. If $A \in F_n$ and $\lambda \in F$, then prove that $tr(\lambda A) = \lambda trA$.

10. If T is unitary and if λ is a characteristic root of T, then prove that $|\lambda| = 1$.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Define the normalizer of a in G. Also prove that $N(a)$ is a subgroup of G.

Or

(b) If p is a prime number and $p^{\alpha} / o(G)$, then prove that G has a subgroup of order p^{α}.

12. (a) Show that every finite abelian group is the direct product of cyclic groups.

Or

(b) Prove that S_n is not solvable for $n \geq 5$.

13. (a) If V is n–dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) Prove that there exists a subspace W of V , invariant under T, such that $V = V_1 \oplus W$.

14. (a) Let $T \in A_F(V)$ has all its distinct characteristic roots, $\lambda_1, \lambda_2, ..., \lambda_k$ in F. Prove that a basis of V can be found in which the matrix T is of the form.

$$\begin{pmatrix} J_1 & & & & \\ & J_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & J_k \end{pmatrix} \text{ where each } J_i = \begin{pmatrix} B_{i1} & & & & \\ & B_{i2} & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & B_{ir_i} \end{pmatrix} \text{ and}$$

where $B_{i1}, B_{i2}, ..., B_{ir_i}$ are basis Jordan blocks belonging to λ_i.

Or

(b) If T in $A_F(V)$ has as minimal polynomial $p(x) = q(x)^2$, where $q(x)$ is a monic, irreducible polynomial in $F[x]$, then prove that a basis of V over F can be found in which the matrix of T is of the form $\begin{pmatrix} C(q(x)^{e_1}) & & & \\ & C(q(x)^{e_2}) & & \\ & & \ddots & \\ & & & C(q(x))^{e_r} \end{pmatrix}$ Where $e = e_1 \geq e_2 \geq ..... \geq e_r$.

15. (a) For all $A, B \in F_n$, prove that

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$

(iii) $(AB)' = B'A'$.

Or

(b) Find the rank and signature of the real quadratic form $x_1^2 + 2x_1x_2 + x_2^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Cauchy theorem.

17. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, ...N_n$. Let $T = N_1 \times N_2 \times ..... \times N_n$. Prove that G and T are isomorphic.

18. Show that two nilpotent linear transformations are similar if and only if they have the same invariants.

19. Prove for each $i = 1, 2, ..., k, V_i \neq (0)$ and $V = V_1 \oplus V_2 ..... \oplus V_{k.}$ Also prove that the minimal polynomial of T_i is $q_i(x)^{li}$.

20. (a) Show that the linear transformation T on V is unitary if and only if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

(b) If N is normal and $AN = NA$, then prove that $AN^* = N^* A$.

S–3874

Sub. Code
23MMA1C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that f is said to be bounded variation on $[a,b]$?

2. Prove that absolute convergence of Σa_n implies convergence.

3. Define a step function.

4. Define the upper and lower stieltjes integral of f with respect ot α.

5. State the first Mean-value theorem for Riemann-Stieltjes integrals.

6. Write the difference between the upper and lower Riemann sums.

7. What is meant by double series?

8. Define the power series. Also find the radius of convergence of $\sum_{n=1}^{\infty} {}^{2^n}\!/_{n}$.

9. When will you say that a sequence of functions $\{f_n\}$ is said to be converge uniformly to f on a set s ?

10. Write short notes on mean convergence with an example.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If f is monotonic on $[a, b]$, then prove that the set of discontinuities of f is countable and also prove f is of bounded variationon $[a,b]$.

Or

(b) Let Σa_n an be an absolutely convergent series having sum s. Prove that every rearrangement of Σa_n an also converges absolutely and has sum s.

12. (a) State and prove Euler's summation formula.

Or

(b) Assume that $\alpha \nearrow$ on [a, b]. If $f \in R(\alpha)$ on $[a,b]$, then prove that $|f| \in R(\alpha)$ on $[a,b]$. Also prove $\left|\int_a^b f(x) d\alpha(x)\right| \le \int_a^b |f(x)| d\alpha(x)$.

13. (a) If f is continuous on $[a,b]$ and if α is of bounded variation on $[a,b]$, then prove that $f \in R(\alpha)$ on $[a,b]$.

Or

(b) State and prove the first fundamental theorem of integral calculus.

14. (a) Assume that $\lim_{p,\, q \to \infty} f(p,q) = 0$. For each fixed p, assume that the limit $\lim_{q \to \infty} f(p,q)$ exists. Prove that the limit $\lim_{p \to \infty}\left(\lim_{q \to \infty} f(p,q)\right)$ also exists and has the value a.

Or

(b) State and prove that Bernstein's theorem.

15. (a) State and prove the Cauchy condition for uniform convergence of series theorem.

Or

(b) Establish Dirichlet's test for uniform convergence theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove.

(a) Dirichlet's test;

(b) Abel's test.

17. If $f \in R(\alpha)$ on $[a,b]$, then prove that $\alpha \in R(f)$ on $[a,b]$ and $\int_a^b f(x) d\alpha(x) + \int_a^b \alpha(x)\, df(x) = f(b)\ \alpha(b) - f(a)\alpha(a)$.

18. Assume that α is of bounded variation on $[a,b]$. Let $V(x)$ denote the total variation of α on $[a, x]$ if $ax \le b$, and let $V(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, then prove that $f \in R(V)$ on $[a,b]$.

19. state and prove Mertens theorem.

20. Assume that each term of $\{fn\}$ is a real-valued function having a finite derivative at each point of an open interval (a,b). Assume that for at least one point x_o in (a,b) the sequence $\{f_n(x_0)\}$ converges. Assume further that there exists a function g such that $f_n' \to g$ uniformly on (a,b).

Prove the following:

(a) There exists a function *f* such that $f_n \to f$ uniformly on (a, b).

(b) For each x in (a,b) the derivative $f'(x)$ exists and equals $g(x)$.

S–3875

Sub. Code
23MMA1C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find all solution of $3y''+2y'=0$.

2. Let $\phi_1(x)=\sin x, \phi_2(x)=e^{ix}$. Determine whether they are linearly dependent or independent.

3. State existence theorem of linear equations with constant coefficients.

4. Using annihilator method find a particular solution of $y''+9y=x^2 e^{3x}$.

5. Write down the Hermite equation.

6. Show that any polynomial of degree n is a linear combination of $p_0, p_1, .. p_n$.

7. Compute the indicial polynomial of $x^2 y''+(x+x^2) y'-y=0$.

8. State the Bessel functions of order α of the first kind.

9. Determine whether the equation $(x^2 + xy)dx + xy\,dy = 0$ is exact.

10. State Lipschitz condition.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$. Show that they are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)\,(x) \neq 0$ for all x in I.

Or

(b) Find the solution of the initial value problem $y'' + 10\,y = 0, y(0) = \pi, y'(0) = \pi^2$.

12. (a) Find four linearly independent solutions of the equation $y^{(4)} + \lambda\,y = 0$, in case : (i) $\lambda = 0$, (ii) $\lambda > 0$, (iii) $\lambda < 0$.

Or

(b) If f, g are two functions with K derivatives then prove that $D^k(fg) = \sum_{l=0}^{k} \binom{k}{l} D^l(f)\,D^{k-l}(g)$, where $\binom{k}{l} = \frac{k!}{(K-l)!\,l!}$.

13. (a) One solution of $x^3 y''' - 3x^2 y'' + 6x y' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) Find two linearly independent power series solutions of the equation $y'' + x^3 y' + x^2 y = 0$.

14. (a) Show that -1 and 1 are regular singular points for the Legendre equation $(1 - x^2) y'' - 2xy' + \alpha(\alpha + 1) y = 0$.

Or

(b) Show that $J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$ is a solution of the equation $x^2 y'' + xy' + x^2 y = 0$.

15. (a) Compute the first four successive approximations ϕ_0, ϕ_1, ϕ_2 and ϕ_3 for $y' = 1 + xy, y(0) = 1$.

Or

(b) State and prove the existence theorem for the convergence of the successive approximations.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Consider the equation $y'' + w^2 y = A \cos wx$, where A, w are positive constants

(a) Find all solutions on $0 \le x < \infty$

(b) Show that every solution ϕ is such that $|\phi(x)|$ assumes arbitrarily large values as $x \to \infty$.

(c) Sketch the graph of that solution ϕ satisfying $\phi(0) = 0, \phi'(0) = 1$.

17. Consider the equation $y^{(5)} - y^{(4)} - y' + y = 0$

(a) Compute five linearly independent solutions

(b) Determine the Wronskian of the solutions found in (a)

(c) Find that solution ϕ satisfying

$\phi() = 1, \phi'(0) = \phi''(0) = \phi'''(0) = \phi^{(iv)}(0) = 0$.

18. (a) Find the solution of legendre equation

$L(y) = (1-x^2)\, y'' - 2x\, y' + \alpha(\alpha+1)\, y = 0$, where α is a constant.

(b) Show that $\int_{-1}^{1} p_n^2(x) dx = 2/2n+1$.

19. Find a basis for the solutions of the second order Euler equation $x^2 y'' + axy' + by = 0$, where a, b are constants and deduce a basis for the solutions of the n[th] order Euler equation on any interval not containing $x = 0$.

20. Prove that necessary and sufficient condition for an equation $M(x, y) + N(x, y)\, y' = 0$ to be exact.

S–3876

Sub. Code
23MMA1E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

Elective – NUMBER THEORY AND CRYPTOGRAPHY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Lucas numbers.

2. Write any two properties of L.C.M.

3. Define composite number.

4. State Dirichlet's theorem.

5. Define multiplication modulo m.

6. State Fermat's little theorem.

7. Find the smallest integer for which $\phi(x) = 6$.

8. Find the value of $\phi(735)$.

9. Define discrete logarithm.

10. What is meant by digraph transformation?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Euclid's Lemma.

Or

(b) Prove that $9 / n$ if and only if $9 / t_{10}(n)$.

12. (a) Let $f(x)$ be a polynomial of degree $r > 1$ with integer coefficients and positive lead coefficients, that is , $f(x) = \sum_{k=0}^{r} a_k x^k$ with $a_r > 0$. Show that there are infinitely many n such that $f(n)$ is composite.

Or

(b) If d and n are natural numbers, then prove that $\left[\frac{n}{d}\right] - \left[\frac{n-1}{d}\right] = \begin{cases} 1 & if \quad d \mid n \\ 0 & if \quad d \nmid n \end{cases}$.

13. (a) Show that $ax \equiv b \pmod{m}$ has at least one solution if and only if $(a, m) \mid b$.

Or

(b) If $m \neq n$, then prove that $(f_m, f_n) = 1$.

14. (a) If f and g are both multiplicative functions, then prove that $f * g$ is also multiplicative function.

Or

(b) Evaluate (i) $\left(\frac{-35}{97}\right)$ (ii) $\left(\frac{71}{73}\right)$.

15. (a) Solve the following system of simultaneous congruence: $2x + 3y \equiv 1 \pmod{26}$, $7x + 8y \equiv 2 \pmod{26}$.

Or

(b) Write down the application of public key cryptography.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove the division algorithm.

17. State and prove the fundamental theorem of arithmetic.

18. State and prove Chinese remainder theorem.

19. State and prove Lemma of Gauss.

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(Z/NZ)$ and set $D = ad - bc$ prove that the following are equivalent:

(a) g.c.d $(D, N) = 1$;

(b) A has an inverse matrix;

(c) If x and y are not both 0 in Z/NZ, then $\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;

(d) A gives a 1-to-1 correspondence of $(Z/NZ)^2$ with itself.

S–3877

Sub. Code
23MMA1E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

Elective – GRAPH THEORY AND APPLICATIONS

(CBCS – 2023 onwards)

Time : Three Hours — Maximum : 75 Marks

Part A — $(10 \times 2 = 20)$

Answer **all** questions

1. When will you say that two graphs are said to be isomorphic? Give an example.

2. Draw all trees with six vertices.

3. What is meant by edge connectivity? Give an example.

4. Write short notes on Konigsberg problem.

5. Find the number of different perfect matching in K_{2n}.

6. What is meant by k-edge-chromatic? Give an example.

7. Determine the value of $r(2, l)$ and $r(k, 2)$.

8. Define k-critical graph.

9. State the four-colour conjecture.

10. When will you say that a digraph is said to be an orientation of G?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) With the usual notations, prove the following:

    (i) $\sum_{v \in V} d(v) = 2\varepsilon$

    (ii) $\delta \leq 2\varepsilon / v \leq \Delta$.

Or

(b) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

12. (a) What is meant by block of a graph? Give an example. Also of G is a block with $v \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, then prove that G is Hamiltonian.

13. (a) Prove that every 3-regular graph without cut edges has a perfect matching.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edges colouring in which both colours are represented at each vertex of degree at least two.

14. (a) If $\delta > 0$, then prove that $\alpha' + \beta' = v$.

Or

(b) Prove or disprove : In a critical graph, no vertex cut is a clique.

15. (a) State and prove Euler's formula for a connected plane graph.

Or

(b) Prove that a diagraph D contains a directed path of length $\chi - 1$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Prove that a graph is bipartite if and only if it contains no odd cycle.

17. With the usual notations, prove that $k \leq k' \leq \delta$.

18. State and prove the Vizing's theorem.

19. Prove the following:

(a) $r(k, l) \leq \binom{k + l - 2}{k - 1}$

(b) $r(k, k) \geq 2^{k/2}$.

20. Show that every planar graph is 5-vertex-colourable.

S–3880

Sub. Code
23MMA1E5

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

Elective — FUZZY SETS AND THEIR APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the support of a fuzzy set.
2. What is meant by fuzzy relation?
3. Define a plausibility measure function.
4. Write the probability distribution function.
5. What are the types of Hartley information?
6. What is meant by measure of dissonance?
7. State the general fuzzy controller modules.
8. Define a finite fuzzy automation.
9. Write short notes on individual decision making.
10. Define the Hamming distance of fuzzy numbers.

Section B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11\. (a) Show that a fuzzy set A on $\mathbb{R}$ is converse if and only if $A(\lambda x_1 + (1-\lambda)x_2) \geq \min \; [A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0,1]$.

Or

(b) Describe the concept of fuzzy ordering relations in detail.

12\. (a) Show that the function Pl determined by equation $Pl(A) = \Sigma m(B)$, for any given basic assignment $B \cap A \neq \phi \;\; m$ is a plausibility measure.

Or

(b) State and prove a necessary and sufficient condition for a belief measured Bel on a finite power set to be a probability measure.

13\. (a) Consider two fuzzy sets, A and B defined on the set of real numbers $X = [0, 4]$ by the membership grade functions $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$. Draw graphs for these functions and their standard classical complements.

Or

(b) Let m_X and m_Y be marginal basic assignments on sets X and Y, respectively and let m be a joint basic assignment on $X \times Y$ such that $m(A \times B)$ = $m_X(A).\ m_Y(B)$ for all $A \in \; P\,(X)$ and $B \in P \;\; (Y)$. Prove that $C(m) = C(m_X) + C(m_Y)$.

14. (a) Narrate the fuzzy neural network in all details.

Or

(b) Describe the concept of fuzzy dynamic system.

15. (a) Explain the multicriteria decision making with an suitable example.

Or

(b) Discuss the concept of multistage decision making in detail.

Section C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Define transitive closure R_T of a relation R.

(b) Find the transitive max-min closure for a fuzzy relation defined by the membership matrix

$$\begin{bmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}.$$

17. Prove : Given a consonant body of evidence $(\mathfrak{F}, m)$, the associated consonant belief and plausibility measures possess the following properties :

(a) $Bel(A \cap B) = \min[Bel(A), Bel(B)]$ for all $A, B \in P(X)$;

(b) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in P(x)$.

18. State and prove the Gibb's inequality.

19. Discuss the main issues involved in the design of a fuzzy controller for stabilizing an inverted pendulum.

20. Let us assume that each individual of a group of eight decision makers has a total preference ordering $P_i \, (i \in \mathbb{N}_8)$ on a set of alternatives $X = \{w, x, y, z\}$ as follows :

$P_1 = \langle w, x, y, z \rangle$, $P_2 = P_5 \langle z, y, x, w \rangle$, $P_3 = P_7 \langle x, w, y, z \rangle$, $P_4 = P_8 = \langle w, z, x, y \rangle$, $P_6 = \langle z, w, x, y \rangle$. Using the membership function $S(x_i, x_j) = \dfrac{N(x_i, x_j)}{n}$ for the fuzzy group preference ordering relation S (where $n = 8$). Find the fuzzy preference relation. Also find a α - cust of this fuzzy relation S, and group level of agreement concerning the social choice denoted by the total ordering $\langle w, z, x, y \rangle$.

S–3881

Sub. Code
23MMA1E6

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms

   (a) Principal disjunctive normal form;

   (b) Principal conjunctive normal form.

2. Define an open statement.

3. Write $p(x) = x^3 - 6x^2 + 11x - 6$ in telescopic form.

4. If a denotes Ackermann's function, evaluate $A(2,1)$ and $A(2,2)$.

5. Draw the Hasse diagram of $p(\{1,2,3,4\},\subseteq)$.

6. Define a Boolean Lattice. Give an example.

7. Draw the Block diagram for coding.

8. Define a group code. Give an example.

9. How many ways are to select five players from a 10 member tennis team to make a trip to a match at another school?

10. If n is a nonnegative integer, then prove that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain the principal conjunctive normal form of the formula $(\neg P \to R) \wedge (Q \leftrightarrow P)$.

Or

(b) Write each of the following in symbolic from (Assume that the universe consists of literally every thing).

(i) All men are giants

(ii) No mean are giants

(iii) Some men are giants

(iv) Some then are not giants

12. (a) Solve $s(k) - 3s(k-1) - 4s(k-2) = 4^k$.

Or

(b) Show that $f(x, y) = x * y$ is a primitive recursive function.

13. (a) Prove that every chain is a distributive lattice.

Or

(b) In a Boolean algebra L, prove that

$(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$ for all $a, b \in L$.

14. (a) Define the hamming distance. State and prove properties of distance function δ.

Or

(b) Let $e: B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a non-zero code word.

15. (a) How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Or

(b) How many ways are there to put four different employees into there indistinguishable offices, when each office can contain any number of employees?

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Is the following conclusion validly derivable form the premises given? If $(\forall x)\,(P(x)) \to Q(x) : (\exists y) P(y)$, then $(\exists z) Q(z)$.

17. Using the generating function solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$ given $y_1 = 1, y_0 = 2$.

18. Simplify the following using karnaugh diagrams $f(x_1, x_2, x_3, x_4) = x_1x_3 + x_1'x_3x_4 + x_2x_3'x_4 + x_2'x_3x_4$.

19. (a) Show that an (m,n) encoding function $e: B^m \to B^n$ can detect k or fever errors if and only if its minimum distance is at least $k+1$.

(b) Let $x = y_1 y_2 \cdots y_m c_1 c_2 \cdots c_r \in B^{m+r}$. Prove that $x * H = 0$ if and only if $x = e_H(b)$ for some $b \in B^m$

20. (a) Generate the permutations of the integers 1, 2, 3 in lexicographic order.

(b) Write down the algorithm for generating the next larger bit string. Also find the next bit string after 1000100111.

S–3882

Sub. Code
23MMA2C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

ADVANCED ALGEBRA

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by finite extension of F ?

2. If a is any real number, prove that $\left(\frac{a^m}{m!}\right) \to 0$ as $m \to \infty$.

3. Define a splitting field.

4. Is any finite extension of a field of characteristic 0 is a simple extension? Justify.

5. Define a normal extension of F.

6. Define the n^{th} cyclotomic polynomial.

7. How many primitive roots does a prime p have?

8. If $t > 1$ is an integer and $\frac{(t^m - 1)}{(t^n - 1)}$, prove that $\frac{m}{n}$.

9. Give an example of a field k algebraic over another field F but not finite-dimensional over F.

10. Find all the elements a in Q_o such that a^{-1} is also in Q_o.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If L is an algebraic extension of k and if k is an algebraic extension of F, then prove that L is an algebraic extension of F.

Or

(b) Using the infinite series for e, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{m!} + ...$, prove that e is irrational.

12. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Or

(b) Prove that $(f(x) + g(x))' = f'(x) + g'(x)$ and $(\alpha f(x))' = \alpha f'(x)$ for $f(x)$, $g(x) \in F[x]$ and $\alpha \in F$.

13. (a) Prove that k is a normal extension of F if and only if k is the splitting field of some polynomial over F.

Or

(b) Express the following as polynomials in the elementary symmetric functions in x_1, x_2, x_3:

(i) $x_1^2 + x_2^2 + x_3^2$

(ii) $(x_1 - x_2)^2 \ (x_1 - x_3)^2 \ (x_2 - x_3)^2$

14. (a) Prove : For every prime number p and every positive integer m there exists a field having p^m elements.

Or

(b) If a is an integer not divisible by the odd prime p, then prove that $x^2 \equiv a \pmod{p}$ is solvable for some integer x if and only if $a^{(p-1)/2} \equiv 1 \pmod{p}$.

15. (a) State the Frobenius theorem. If A is a ring algebraic over a field F and A has no zero divisors, then prove that A is a division ring.

Or

(b) State the Lagrange identity. If $a \in H$ then prove that $a^{-1} \in H$ if and only if $N(a) = 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If L is a finite extension of k and if k is a finite extension of F, then prove that L is a finite extension of F. Also prove $[L:F] = [L:K]\ [K:F]$.

17. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

18. State and prove the fundamental theorem of Galois theory.

19. State and prove the Wedderburn theorem.

20. Establish left-division algorithm.

S–3883

Sub. Code
23MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Show that outer measure is translation invariant.

2. Prove that the constant functions are measurable.

3. Show that if f is non-negative measurable function, then $f = 0$ a.e if and only if , $\int f dx = 0$.

4. Show that if f is integrable, than f is finite-valued a.e.

5. Define orthonormal on I .

6. State Dini's test.

7. Write the first-order Taylor formula.

8. Define Jacobian matrix.

9. Prove that if $f = u + iv$ is a complex-valued function with a derivative at a point z in C, then $J_f(z) = |f'(z)|^2$.

10. State the implicit function theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that, for any sequence of sets $\{E_i\}$, $m^*\left(\bigcup_{i=1} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$.

Or

(b) Prove that not every measurable set is a Borel set.

12. (a) State and prove Lebesgue's Monotone convergnce theorem.

Or

(b) Show that $\lim \int_0^{\infty} \frac{dx}{(1 + x/n)^n x^{1/n}} = 1$.

13. (a) State and prove Jordan's theorem.

Or

(b) Assume that $f \in L([0, 2\pi])$ and suppose that f is periodic with period 2π. Let $\{S_n\}$ denote the sequence of partial sums of the Fourier series generated by f, say $s_n(x) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)$, $n = 1, 2, \ldots$. Then prove that $s_n(x) = \frac{2}{\pi} \int_0^{\pi} \frac{f(x+t) - f(x-t)}{2} D_n(t) dt$.

14. (a) State and prove mean-value theorem.

Or

(b) State and prove Taylor's formula.

15. (a) Let A be an open subset of $\mathbb{R}^n$ and assume that $f : A \to \mathbb{R}^n$ has continuous partial derivatives $D_j f_i$ on A if $J_f(x) \neq 0$ for all x in A, than prove that f is an open mapping.

Or

(b) State and prove second-derivative test for extrema.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the outer measure of an internal equals its length.

17. Let f the bounded and measurable on a finite interval $[a,b]$ and let $\in > 0$. Then prove that

There exist

(a) a stop function h such that $\int_a^b |f - h| dx \leq \in$.

(b) a continuous function g such that g vanishes outside a finite interval and $\int_a^b |f - g| dx < \in$.

18. State and prove Fejer's theorem.

19. State and prove chain rule theorem.

20. State and prove the inverse function theorem.

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S–3884

Sub. Code
23MMA2C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Classify the partial differential equation $U_{xx} + x^2 U_{yy} = 0$.

2. Write the partial differential equation for the telegraph equation.

3. State Cauchy-Kowaleswky theorem.

4. Define domain of influence and domain of dependence.

5. Mention the heat conduction problem as the IVP with suitable conditions.

6. State the uniqueness theorem on one-dimensional wave equation.

7. Define Harmonic function.

8. Prove that the solution of the Dirichlet problem if it exists is unique.

9. Define dirac delta function in three dimensional problem.

10. Define Green's function.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) What are the assumptions made by deriving the one-dimensional wave equation?

Or

(b) Obtain the general solution of the PDE $3\,u_{xx} + 10\,u_{xy} + 3\,u_{yy} = 0$.

12. (a) Determine the solution of the Initial value problem $u_{tt} = 4\,u_{xx},\ x > 0,\ t > 0;$ $u(x, 0) = |\sin x|;\ x > 0$ $u_t(x, 0) = 0,\ x \geq 0$ and $u(x, 0) = 0,\ t \geq 0$.

Or

(b) Find the solution of the following IVP's: $u_{tt} = C^2\,u_{xx}$; $u(x, 0) = \sin x;\ u_t(x, 0) = \cos x$.

13. (a) Explain the plucked string problem for finding the solution of the vibration of a stretched string.

Or

(b) Solve the Eigen value problem $X'' - \lambda X = 0$; $X(0) = 0$ and $X(l) = 0$.

14. (a) State and prove the Minimum principle theorem.

Or

(b) Discuss about the solution of the Dirichlet problem for circular annulus.

15. (a) Show that the Green's function is symmetric.

Or

(b) Find the Free-space Green's function of the Laplace operator.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Find the characteristic equations and characteristics and then reduce the equations $u_{xx} \mp [\sec h^4 x]\, u_{yy} = 0$ to the canonical form.

17. Derive the D'Alembert solution of the Cauchy problem for the one-dimensional wave equation.

18. State and prove the uniqueness theorem of the solution of the one-dimensional heat equation.

19. Establish a necessary and sufficient condition for the existence of a solution for the Interior Neumann problem $\nabla^2 u = 0; r < R, \frac{\partial u}{\partial n} = \frac{\partial u}{\partial r} = f(\theta)$ on $r = R$.

20. Solve the Neumann problem $\nabla^2 u + k^2 u = h$ in R $\frac{\partial u}{\partial n} = 0$ on S.

S–3886

Sub. Code
23MMA2E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Second Semester

Mathematics

Elective – MATHEMATICAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find the intersection $c_1 \cap c_2$ of the two sets c_1 and c_2 where $c_1 = \{(x, y); 0 < x < 2, 1 < y < 2\}$, $c_2 = \{(x, y); 1 < x < 3, 1 < y < 3\}$.

2. State the Baye's theorem.

3. Let X be the random variable of the uniform distribution with pdf $f_X(x) = \frac{1}{2a}, -a < x < a$, zero elsewhere. Find the mean and variance of X.

4. State the Jensen's inequality.

5. Suppose the probability that a person has blood type B is 0.12. In order to conduct a study concerning people with blood type B, patients are sampled independently of one another until 10 are obtained who have blood type B. Determine the probability that at most 30 patients have to have their blood type determined.

6. Suppose that 10% of the probability for a certain distribution that is $N(\mu, \sigma^2)$ is below 60 and that 5% is above 90. What are the values of μ and σ?

7. Let $\overline{X}$ be the mean of a random sample of size n from a distribution that is $N(\mu, 9)$. Find n such that $P(\overline{X} - 1 < \mu < \overline{X} + 1) = 0.90$, approximately.

8. Define the statistic.

9. What is meant by converges in probability?

10. Define t-distribution.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the Poker Hands problem.

Or

(b) Bowl I contains six red chips and four blue chips. Five of there 10 chips are selected at random and without replacement and put in bowl II, which war originally empty, one chip is then drawn at random from bowl II. Given that this chip is blue, find the conditional probability that two red chips and three blue chips are transferred from bowl I to bowl II.

12. (a) Let x have the pdf $f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$ Find $E(X)$, $E(X^2)$ and $E(6X + 3X^2)$.

Or

(b) State and prove the Markov's inequality.

13. (a) If the random variable x is $N(\mu, \sigma^2), \sigma^2 > 0$, then prove that the random variable $v = \frac{(x-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

Or

(b) Derive the p.d.f of Dirichlet distribution.

14. (a) Suppose $X_n \xrightarrow{P} a$ and the real function g is continuous at 'a'. Then prove that $g(X_n) \xrightarrow{P} g(a)$.

Or

(b) If X_n converges to X in probability, then prove that X_n converges to X in distribution.

15. (a) Suppose the number of customers X that enter a store between the hours 9:00 a.m and 10:00 a.m follows a poisson distribution with parameter θ. Suppose a random sample of the number of customers that enter the store between 9:00 a.m and 10:00 for 10 days results in the values

9 7 9 15 10 13 7 2 12

Determine the maximum likelihood estimator of θ. Show that it is an unbiased estimator.

Or

(b) Explain the zea mays data probelm.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that for any random variable, $P[X = x] = F_X(x) - F_X(-x)$ for all $x \in R$, where $F_X(x-) = \lim_{z \uparrow x} F_X(z)$.

(b) Let x be a continuous random variable with pdf $f_X(x)$ and support S_X. Let $Y = g(X)$, where $g(x)$ is a one-to-one differentiable function, on the support of X, S_X. Denote the inverse of g by $x = g^{-1}(y)$ and let $\frac{dx}{dy} = \frac{d[g^{-1}(y)]}{dy}$. Then prove that the pdf of Y is given by $f_Y(y) = f_X(g^{-1}(y))\left|\frac{dx}{dy}\right|$, for $y \in S_Y$, where the support of Y is the set $S_Y = \{y = g(x); x \in S_X\}$.

17. Let (X_1, X_2) be a random vector such that the variance of x_2 is finite, Then prove that

(a) $E[E(X_2 \mid X_1)] = E(X_2)$

(b) $\text{var}[E(X_2 \mid X_1)] \le \text{var}(X_2)$

18. Compute the measures of Skewness and Kurtosis of the poisson distribution with mean μ.

19. State and prove the central limit theorem.

20. A die was cast $n = 120$ independent times and the following data resulted.

Sports up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40-b

If we use a chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at the 0.025 significance level?

S–3888

Sub. Code
23MMA2E4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Second Semester

Mathematics

Elective – CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define variation of the functional.

2. State Euler's equations.

3. State the transversality condition.

4. Write short notes on strong extremum.

5. State the reciprocity principle.

6. Write the ostrogradsky equation.

7. Define symmetric keruel.

8. State Fredholm theorem.

9. Write down the volterra integral equation.

10. State the Fredholm's second theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the extremals of the functional

$$V(y(x)) = \int_{x_0}^{x_1} y'(1 + x^2 y')dx .$$

Or

(b) Write the ostrogradsky equation for the functional $V(z(x,y)) = \iint_D \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial 4}\right)^2 dx\,dy$.

12. (a) Find the transversality condition for the functional $V = \int_{x_0}^{x_1} A(x,y,z)\sqrt{1 + y'^2 + z'^2}\,dx$ if $z_1 = \varphi(x_1, y_1)$.

Or

(b) Test for an extremum the functional

$$V(y(x)) = \int_0^a y'^2 dx;\ y(0) = 0,\ y(a) = b, a \geq 0, b \geq 0 .$$

13. (a) Find an approximate solution of the problem of the minimum of the functional

$$V(y(x)) = \int_0^1 (y'^2 - y^2 - 2xy)dx, y(0) = y(1) = 0$$

Or

(b) Find the extremal of the functional $V(y(x)) = \int_0^{\pi} \left(y'^2 - y^2\right) dx$ under the conditions $y(0) = 0, y(\pi) = 1$ and subject to the constraint $\int_0^{\pi} y\, dx = 1$.

14. (a) Find the eigen values and eigen functions of the homogenous integral equation.

$$y(x) = \lambda \int_0^1 \left(2xt - 4x^2\right) y(t)\, dt$$

Or

(b) Solve the homogenous Fredholm integral equation $g(s) = \lambda \int_0^1 e^{SHt} g(t)\, dt$.

15. (a) Solve the volterra equation $g(s) = 1 + st \int_0^s st\, g(t)\, dt$.

Or

(b) Solve the homogenous equations $g(s) = \frac{1}{2} \int_0^{\pi} (\sin(s+t)) g(t)\, dt$ by using the explicit formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove fundamental lemma of calculus of variations.

(b) Derive Euler's equation.

17. Test for an extremum the functional $v = \int_{x_0}^{x_1} \left(y'^2 + z'^2 + 2zy\right)dx$ given that $y(0) = 0;\ z(0) = 0$ and the point (x_1, y_1, z_1) can move over the plane $x = x_1$.

18. Using the Ritz method, find an approximate solution of the differential equation $y'' + x^2 y = x\ \ y(0) = y(1) = 0$. Determine $y_2(x)$ and $y_3(x)$ and compare their values at the points $x = 0.25,\ x = 0.5$ and $x = 0.75$.

19. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s + t) g(t) dt$.

20. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (1 - 3st) g(t) dt$ and evaluate the resolvent kernel.

S–3890

Sub. Code
23MMA2E6

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

Elective – MACHINE LEARNING AND ARTIFICIAL INTELLIGENCE

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions

1. Define general boundary and specific boundary.

2. How do you design a checkers learning problem?

3. What is Artificial Neural Network?

4. Under what condition the perceptron rule fails and it becomes necessary to apply the delta rule?

5. What are Bayesian Belief nets?

6. Define conditional probability.

7. What is adversarial search?

8. Define artificial intelligence.

9. What are the different types of planers?

10. State Baye's rule.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Explain the inductive biased hypothesis space and unbiased learner.

Or

(b) What are the issues in machine learning?

12. (a) Differentiate between Gradient descent and stochastic gradient descent.

Or

(b) Brief the origin of genetic algorithm.

13. (a) Explain brute force Baye's concept learning.

Or

(b) Discuss about Naive Baye's classifier with an example.

14. (a) Explain the role of an agent program.

Or

(b) What are the applications of artificial intelligence?

15. (a) Write forward (progression) state-space search algorithm.

Or

(b) Explain about the exact inference in Bayesian networks.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain the list then eliminate algorithm with an example.

17. Derive the backprobagation rule considering the training rule for output unit weights and training rule for Hidden unit weights.

18. Enumerate Bayesian belief network and conditional independence with an example.

19. Narrate the alpha-beta pruning.

20. Describe the hierarchical planning method with an example.

S–3891

Sub. Code
23MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define the index of the point a with respect to the curve γ.

2. State the fundamental theorem of algebra.

3. Define homologous to zero.

4. Find the residues of $\dfrac{e^z}{(z-a)(z-b)}$ at its poles.

5. Define a potential function.

6. Write down the poisson's formula in polar coordinates.

7. If $f(z)$ is analytic in the whole plane and real an the real axis, purely imaginary on the imaginary axis, show that $f(z)$ is odd.

8. Obtain the series expansion for are *tan z* and are *sin z*.

9. Define genus of a canonical product.

10. Define $\sqrt{(z)}$ and find $\sqrt{\left(\frac{1}{2}\right)}$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove representation formula. Also find $\int\limits_{|z|=2} \frac{dz}{z^2+1}$.

Or

(b) State and prove Weierstrass theorem an essential singularity.

12. (a) State and prove the residues theorem.

Or

(b) State and prove the Rouche's theorem.

13. (a) Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}, a>1$.

Or

(b) If u_1 and u_2 are harmonic in a region Ω, then prove that $\int\limits_{\gamma} u_1\,{}^*du_2 - u_2\,{}^*du_1 = 0$.

14. (a) State and prove Hurwitz theorem.

Or

(b) State the Laurent series. Also prove that the Laurent development is unique.

15. (a) Prove the following:

(i) $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$;

(ii) $(1+z)(1+z^2)(1+z^4)\,(1+z^8)\cdots=\frac{1}{1-z}$ if $|z|<1$.

Or

(b) With the usual notations, prove that $\sqrt{(z)}\,\sqrt{(1-z)}=\frac{\pi}{\sin \pi z}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the local mapping theorem.

17. State and prove the general from of Cauchy's theorem.

18. Derive the Poisson's formula.

19. Establish the Schwarz's theorem.

20. Derive the Jensen's formula.

S–3892

Sub. Code
23MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

PROBABILITY THEORY

(CBCS – 2023 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by Borel field of events?

2. Prove that the probability of the impossible event is zero.

3. Define symmetric distribution.

4. The random variable X is of the continuous type with the density defined as $f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \cos x & \text{for} \quad 0 \le x \le \frac{\pi}{2} \\ 0 & \text{for} \quad x > \frac{\pi}{2} \end{cases}$.

   Find the median of X.

5. Show that the random variables X and Y are dependent where the joint distribution

$$f(x, y) = \begin{cases} \frac{1}{4}[1 + xy(x^2 - y^2)] & \text{for} \quad |x| \le 1 \text{ and } |y| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

6. Write down the necessary and sufficient conditions for a function $\phi(t)$ to be a characteristic function.

7. What is meant by a poly a distribution?

8. Define a Beta distribution.

9. Define stochastically convergent.

10. State the Borel-Cantelli Lemma.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Three dice are thrown once. Compute the probability of obtaining.

    (i) Face 3 on at least one die

    (ii) A sum exceeding 7

    (iii) A sum which is a prime number.

Or

(b) Let $\{A_n\}, n = 1, 2, \ldots,$ be a non decreasing sequence of events and let A be their alternative. Prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

12. (a) If a random variable Y can take on only non-negative values and has expected value $E(Y)$, then prove that for an arbitrary positive number K, $P(Y \geq K) \leq \frac{E(Y)}{K}$.

Or

(b) Prove that the coefficient of correlation satisfies the double inequality $-1 \leq \rho \leq 1$.

13. (a) Show that the distribution function $F(x,y)$ of a two-dimensional random variables (x,y) is uniquely determined by the class of all one-dimensional distribution functions of $tx+uy$, where t and u run over all possible real values.

Or

(b) Find the characteristic functions of the random variables whose densities is $f(x)=\frac{2\sin^2\left(\frac{ax}{2}\right)}{\pi a x^2}$.

14. (a) Show that if the random variables X_1 and X_2 have zero-one distributions and are uncorrelated, they are independent.

Or

(b) State and prove the Poisson's theorem.

15. (a) Show that if the sequence of characteristic function $\{\phi_n(t)\}$ converges to the characteristic function $\phi(t)$ and $t_n \to t_0$, then $\phi_n(t_n) \to \phi(t_0)$.

Or

(b) Let $\{x_k\},(k=1,2,....)$ be a sequence of independent random variables with the same distribution and with expected value $E(x_k)=m$. Then prove that the sequence $\{y_n\}$, where $y_n=\frac{x_1+x_2+...+x_n}{n}$, is stochastically convergent to m.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. The distribution of the random vector (x, y) is given by the formulas

$$p(x=1, y=1) = p(x=1, y=2) = p(x=2, y=2) = \frac{1}{3}.$$

(a) Find the distribution functions $F(x, y), F_1(x)$ and $F_2(y)$

(b) Check whether the points $\left(1, \frac{1}{2}\right), (1,3), \left(2, \frac{1}{2}\right)$ and $(2,3)$ are discontinuity points of $F(x, y)$.

17. Show that if ρ is the correlation coefficient of the random variables x_1 and x_2, ρ is also the coefficient of correlation of the random variables $y_i = a_i + x_i + b (i = 1, 2)$.

18. The joint distribution of the random variable (x, y) is given by the density

$$f(x, y) = \begin{cases} \frac{1}{4}[1 + xy(x^2 - y^2)] & for \quad |x| \le 1 \; and \; |y| \le 1 \\ 0 & for \quad \text{all other points} \end{cases}$$

(a) Show that the random variables x and y are dependent.

(b) Find the density of the sum $z = x + y$.

(c) Determine the characteristics function of x, y and $z = x + y$.

19. Derive the characteristic function of a Cauchy distribution.

20. State and prove the de Moivre-Laplace theorem.

S–3893

Sub. Code
23MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the product topology.

2. What is meant by order topology? Give an example.

3. Write short notes on $\in - \delta$ definition of continuity by giving an example.

4. State the sequence lemma.

5. When will you say a topological space is connected?

6. Define punctured Euclidean space.

7. Prove or disprove : The interval [0, 1] is not compact.

8. Define a Lebesgue number.

9. Is the product of two Lindelof spaces need not be Lindelof? Justify your answer.

10. Define normal space with an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that the collection

$S = \{\pi_1^{-1}(U) \mid U$ open in $X\}$ $U\{\pi_2^{-1}(V) \mid V$ open in $Y\}$ is a subbasis for the product topology on $X \times Y$

Or

(b) Let X be a topological space. Prove that the following conditions hold :

(i) ϕ and X are closed;

(ii) Arbitrary intersection of closed sets are closed

(iii) Finite unions of closed sets are closed.

12. (a) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.

Or

(b) Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

13. (a) State and prove intermediate value theorem.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

14. (a) Let $f : X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that f is a homeomorphism.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Prove that every metrizable space is normal.

Or

(b) Define completely regular space. Also prove that a product of completely regular spaces is completely regular.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' respectively on X. Prove the following are equivalent :

(i) τ' is finer than τ

(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

(b) Show that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are not comparable.

17. Let X and Y be topological spaces and let $f : X \to Y$. Prove the following are equivalent :

   (a) f is continuous

   (b) For every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$

   (c) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X

   (d) For every $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

18. (a) Prove that the image of a connected space under a continuous map is connected.

   (b) Show that a finite Cartesian product of connected spaces is connected.

19. (a) State and prove Extreme value theorem.

   (b) State and prove Uniform continuity theorem.

20. Prove that every regular space X with a countable basis is metrizable.

S–3894

Sub. Code
23MMA3C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Third Semester

Mathematics

INDUSTRIAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the confidence internal for μ when σ is known in $N(\mu,\sigma^2)$.

2. Define a power function.

3. Define sufficient statistics for parameter θ.

4. Distinguish between completeness and uniqueness.

5. Define an efficient estimator of θ. Give an example.

6. What is meant by robust estimator?

7. Define a best critical ragion of size α.

8. What is the importance of likelihood ratio test?

9. What is analysis of variance?

10. Define the correlation coefficient of the random sample.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11\. (a) Let $X_1, X_2, ...X_n$ denote a random sample from a distribution having the following probability density function $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$

$= 0$ elsewhere.

Find the maximum likelihood estimator $\hat{\theta}$ of θ.

Or

(b) Enumerate the independence of attributes rising Chi-square test.

12\. (a) Show that the mean $\overline{X}$ of a random sample of size n from a distribution having $p.d.f$ $f(x;\theta) = \frac{1}{\theta} e^{-(x/\theta)}$, $0 < x < \infty$, $0 < \theta < \infty$, zero elsewhere, is an unbiased estimator of θ and has variance θ^2/n.

Or

(b) Let $X_1, X_2, ...X_n$ denote a random sample from a distribution that is $N(0,\theta)$. Prove that $y = \sum X_i^2$ is a complete sufficient statistic for θ . Find the unbiased minimum variance estimator of θ^2 .

13\. (a) Discuss the Bayesian estimation.

Or

(b) Let $X_1, X_2, ...X_n$ denote a random sample from the distribution $b(1,\theta)$, $0 \le \theta \le 1$. Find the maximum likelihood estimate $\hat{\theta}$ of θ and $\text{var}(\hat{\theta})$.

14. (a) Let $X_1, X_2, \ldots . X_{10}$ be a random sample of size 10 from a normal distribution $N(0, \sigma^2)$. Find a best critical region of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$.

Or

(b) Let X be $N(\theta, 100)$, find the sequential probability ratio test for testing $H_0 : \theta = 75$ against $H_1 : \theta = 78$.

15. (a) With the usual notations, prove that $Q = Q_2 + Q_4 + Q_5$.

Or

(b) Find the moment generating function of a noncentral Chi-square distribution.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain the procedures for the construction of confidence intervals for

(a) Means and

(b) Difference of means.

17. State and prove the Neymann factorization theorem.

18. (a) Derive the Rao – Cramer inequality.

(b) Determine the Rao – Cramer lower bound in the case of $f(x;\theta) = \frac{1}{\pi[1 + (x - \theta)^2]}, -\infty < x < \infty$, $-\infty < \theta < \infty$.

19. Let $X_1, X_2, \ldots. X_n$ and $Y_1, Y_2, \ldots. Y_m$ be independent random sample from the distributions $N(\theta_1, \theta_3)$ and $N(\theta_2, \theta_{4)}$, respectively. Show that the likelihood ratio for testing $H_0 : \theta_1 = \theta_2$, $\theta_3 = \theta_4$ against all alternatives is given by

$$\frac{\left[\sum_1^n (x_i - \overline{x})^2 / n\right]^{n/2} \left[\sum_1^m (y_i - \overline{y})^2 / m\right]^{m/2}}{\left\{\left[\sum_1^n (x_i - u) + \sum_1^m (y_i - u)^2\right] / (m+n)\right\}^{(n+m)/2}} ,$$ where

$u = (n\overline{x} + m\overline{y}) / (n+m)$.

20. (a) Discuss the role of the distributions of a certain quadratic forms in the techniques of analysis of variance.

(b) Show that $R = \dfrac{\sum_1^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_1^n (X_i - \overline{X})^2 \sum_1^n (Y_i - \overline{Y})^2}}$

$$= \frac{\sum_1^n X_i Y_i - n\overline{X}\,\overline{Y}}{\sqrt{\left(\sum_1^n X_i^2 - n\overline{X}^2\right)\left(\sum_1^n Y_1^2 - n\overline{Y}^2\right)}} .$$

S–3895

Sub. Code
23MMA3E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

Elective – ALGEBRAIC NUMBER THEORY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define R-module.

2. Define symmetric polynomials.

3. Express $Q\left(\sqrt{3}, \sqrt[3]{5}\right)$ in the form $Q(\theta)$.

4. Define algebraic integer.

5. Define integral basis.

6. Find the ring of integers of $Q\left(\sqrt[3]{175}\right)$.

7. Define a quadratic field.

8. Define a cyclotomic field.

9. State Fermat theorem.

10. What is meant by factorization into irreducibles?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that every finite integral domain is a field.

Or

(b) Let G be a free abelian group of rank n with basis $\{x_1, x_2, ... x_n\}$. Suppose (a_{ij}) is an $n \times n$ matrix with integer entries. Then prove that the elements $y_i = \sum_j a_{ij} x_j$ from a basis of G iff (a_{ij}) is unimodular.

12. (a) Show that the co-efficients of the field polynomial are rational numbers, so that $f_\alpha(t) \in Q[t]$.

Or

(b) Prove that the algebraic integers from a subring of the field of algebraic numbers.

13. (a) If $\{\alpha_1, \alpha_2 .... \alpha_n\}$ is a basis of K consisting of integers, then prove that the discriminant $\Delta[\alpha_1, \alpha_2 ...., \alpha_n]$ is a rational integer, not equal to zero.

Or

(b) Find the ring of integers of $Q(\sqrt{2}, i)$.

14. (a) Show that the minimum polynomial of $(\zeta) = e^{2\pi i/p}$, p an odd prime, over Q is $f(t) = t^{p-1} + t^{p-2} + .. + t + 1$. The degree of $Q(\zeta)$ is $p-1$.

Or

(b) Let d be a α square free rational integer. Then prove that the integers of $Q(\sqrt{2})$ are

(i) $z[\sqrt{d}]$ if $d \not\equiv = 1 \pmod 4$

(ii) $z\left[\frac{1}{2} + \frac{1}{2}\sqrt{d}\right]$ if $d \not\equiv 1 \pmod 4$

15. (a) If D is a domain and x, y are non-zero elements of D. Then prove that

(i) x / y if $\langle x \rangle \supseteq \langle y \rangle$

(ii) x and y are associates if $\langle x \rangle = \langle y \rangle$.

Or

(b) Show that ring of integers D in a number field K is noetherion.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If $H \subseteq K \subseteq L$ are fields, then prove that $[L:H] = [L:K][K:H]$.

17. If K is a number field then prove that $K = Q(\theta)$ for some algebraic number Q.

18. Let $K = Q(\theta)$ be a number field where θ has minimum polynomial p of degree n. Show that the Q-basis $\{1, \theta, ,....\theta^{n-1}\}$ has discriminant $\Delta[1, ...\theta^{n-1}] = (-1)^{n(n-1)/2} N(DP(\theta))$, where Dp is the formula derivative of p.

19. Prove that the ring D of integers of $Q(\zeta)$ is $z[\zeta]$.

20. Let D be the ring of integers in a number field K, and let $x, y \in D$. Then prove that

(a) x is a unit iff $N(x) = \pm 1$.

(b) If x and y are associates, then $N(x) = \pm N(y)$.

(c) If $N(x)$ is a rational prime, then x is irreducible in D.

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S–3896

Sub. Code
23MMA3E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

Elective – FLUID DYNAMICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by fluid dynamics?

2. Define an equipotentials.

3. Define the hydrostatic pressure.

4. Write down the Bernoulli's equation.

5. What is meant by the vector moment of the doublet?

6. Define an image system of the region.

7. Write down the Cauchy-Riemann equations.

8. Define a line source.

9. What is meant by the co-efficient of viscosity?

10. Define direct stresses.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive the general equation of continuity.

Or

(b) Narrate the acceleration of a fluid.

12. (a) Establish the Euler's equation of motion.

Or

(b) Discuss the case of steady motion under conservative body forces.

13. (a) Show that the velocity at P is $(2ml\cos\alpha)/(a^2 - l^2)^{\vec{u}}$ where $\vec{u}$ is the unit vector along the normal to the spheroid at P and $2\alpha = \underline{|APB}$.

Or

(b) Explain the following terms of the stream function.

(i) Uniform Line source along $\overline{OZ}$;

(ii) Doublet in uniform stream.

14. (a) Discuss the flow due to a uniform line doublet at O of strength μ per unit length, its axis being along $\overline{OX}$.

Or

(b) Find the equations of the streamlines due to uniform line sources of strength m through the points $A(-C, 0), B(C, 0)$ and a uniform line sink of strength $2m$ through the origin.

15. (a) Derive the Navier-Stokes equations of motion.

Or

(b) Show that the velocity profile between the plates is parabolic.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Test whether the motion specified by $\vec{q} = \frac{k^2\left(x\vec{j} - y\vec{i}\right)}{x^2 + y^2}$, ($k$-constant) is a possible motion for an incompressible fluid. If so, determine the equations of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.

17. Discuss the underwater explosion giving spherical gas bubble problem.

18. State and prove the Weiss's sphere theorem.

19. (a) State and prove the Milne-Thomson circle theorem.

(b) Enumerate the uniform flow past a stationary cylinder.

20. Discuss in detail about the relations between Cartesian components of stress.

S–3897

Sub. Code
23MMA3E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

Elective : STOCHASTIC PROCESSES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define weakly stationary.

2. What is transition probability matrix?

3. State Basic limit theorem of renewal.

4. Define null persistent.

5. Define point process.

6. Write down the Erlang's formula.

7. State Renewal theorem.

8. Define a stopping time.

9. What is Bienayame-Galton-Watson process?

10. State Yaglom's theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If the process $X(t) = A_1 + A_2 t$, Where A_1, A_2 are independent random variables with $E(A_1) = \alpha i$, $\text{var}(A_i) = \sigma_i^2, i = 1,2$. Then prove that $\{X(t), t \geq 0\}$ is evolutionary.

Or

(b) Explain the Markov-Bernoulli chain.

12. (a) Show that the state j is persistent iff $\sum_{n=0}^{\infty} p_{j_j}^{(n)} = \infty$.

Or

(b) If state j is persistent non-null, then prove the following as $n \to \infty$.

(i) $p_{jj}^{(nt)} \to t / \mu_{jj}$, when state j is periodic with period t.

(ii) $p_{jj}^{(n)} \to 1 / \mu_{jj}$, when state j is a periodic.

13. (a) State and prove additive property of Poisson process.

Or

(b) Derive Yule-Furry Process.

14. (a) If $p_n \to \alpha$, then prove that $v_n \to \alpha b$, where $b = B(1) = \in b_n$.

Or

(b) State and prove wald's equation.

15. (a) Show that the $p.g.f$ $R_n(s)$ of y_n satisfies the recurrence relation $R_n(S) = SP(R_{n-1}(S))$, $P(S)$ being the $p.g.f$ of the offspring distribution.

Or

(b) Prove that $\frac{\partial}{\partial t} F(t, s) = u(s) \frac{\partial}{\partial s} F(t, s)$ and $\frac{\partial}{\partial t} F(t, s) = u(F(t, s))$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Derive Chapman-Kolmogorov equation.

17. Let $\{X_{n,}\ n \geq 0\}$ be a markov chain having state space $S = \{1, 2, 3, 4\}$ and transition matrix

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Show that state 1 and 2 are ergodic.

18. Illustrate Birth-death process.

19. State and prove Elementary renewal theorem.

20. Prove that $P_n(S) = P_{n-1}(P(s))$ and $P_n(s) = P(p_{n-1}(s))$.

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